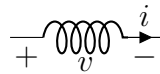The Antoniou Inductance-Simulation Circuit Derivation
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Exactly how does the Inductance-Simulation Circuit work like an inductor? We can start by asking, how should it work?. From an electrical circuit point of view, what properties make an inductor an inductor?

The key is how it works in terms of an electrical circuit! As we know, an inductor can be described by referring to two domains of energy: electrical and magnetic. In this case, we are not trying to use the magnetic fields the inductor creates; we only care about current to voltage relationship in the circuit that the inductor used in. This would be a different conversation if we were trying to use the inductor as a transformer, or in any other application that uses an inductor's magnetic field.
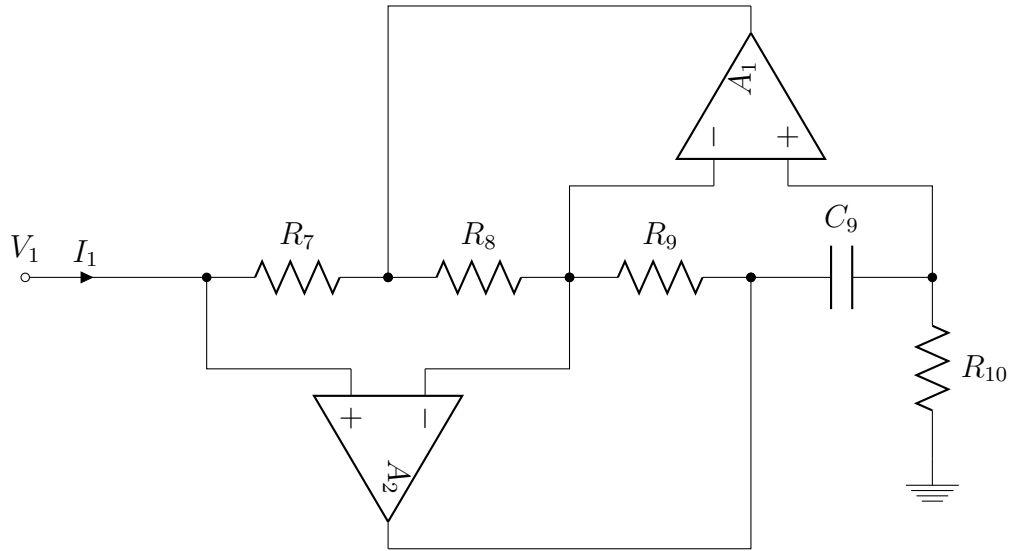
The current to voltage relationship of the inductor is

$$v(t) = L\frac{di(t)}{dt} \tag{1}$$

To make thing's clear, this equation is our goal. We want to make a circuit that has this relationship at the terminals. If we achieve this, then we are simulating an inductor.

We will use the Antoniou Inductance-Simulation Circuit as our prime example.

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The point is that we should be able to use this circuit in place of an inductor. While it may or may not be obvious, we don't particularly care what's going on inside the Antoniou Circuit as long as the terminal relationship (the relationship between V_1 and I_1) is the same as an inductor.

i.e. if we can show that

$$V_1(t) = L_1 \frac{dI_1(t)}{dt}.$$ (2)

Before we go any further, I would like to express the terminal relationship of an inductor in a slightly different fashion by applying the Laplace transform. We can say that current to voltage relationship of the inductor is

$$\frac{V(s)}{I(s)} = Ls$$ (3)

So now if we show that the Antoniou Circuit has a terminal relationship of

$$\frac{V_1(s)}{I_1(s)} = L_1 s$$ (4)

then it simulates the inductor.

How would we begin to show the terminal relationship of the Antoniou Circuit?

The concept is simple: using very basic theorems in all the right places and some algebra, we must get the relationship between V_1 and I_1.

Let's start with something simple, what are the voltages at the input terminals of the IDEAL op-amps?
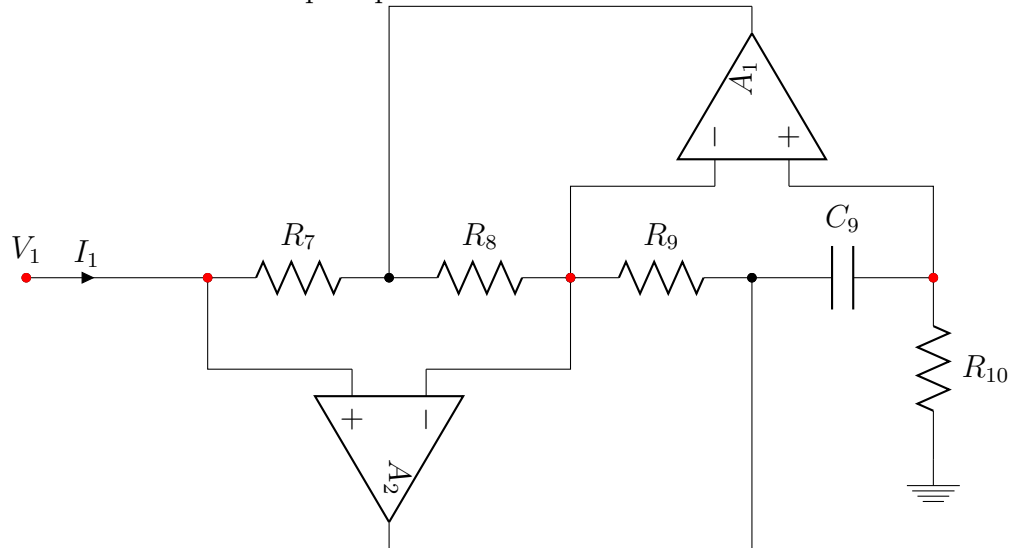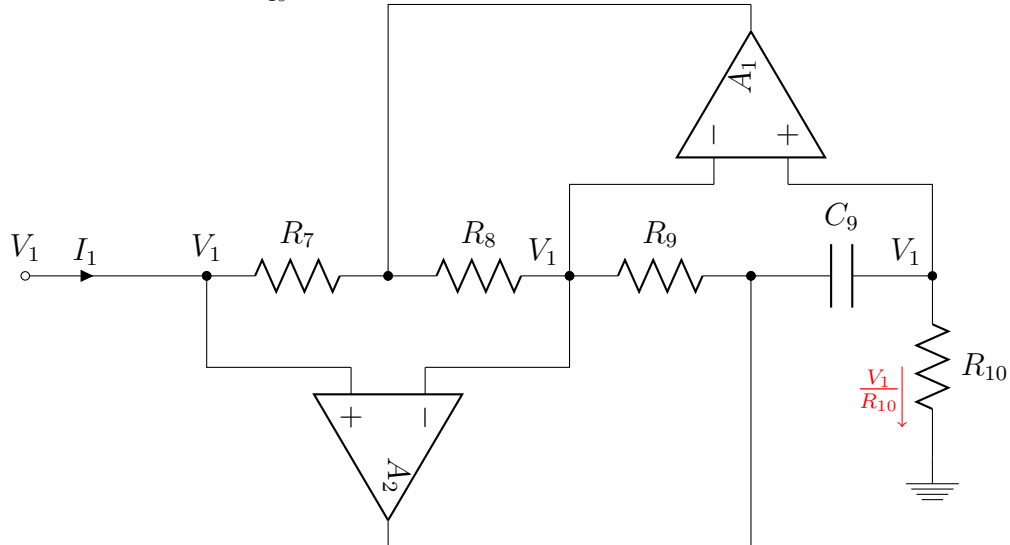

From the ideal op-amp model we know that the voltage at the + and - terminal of an op-amp is the same. Knowing this we can say that:
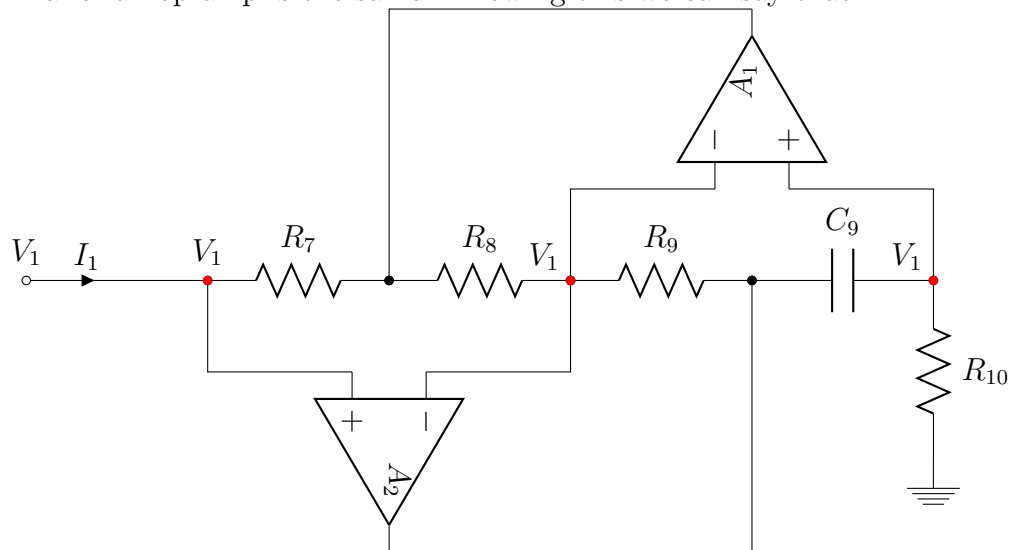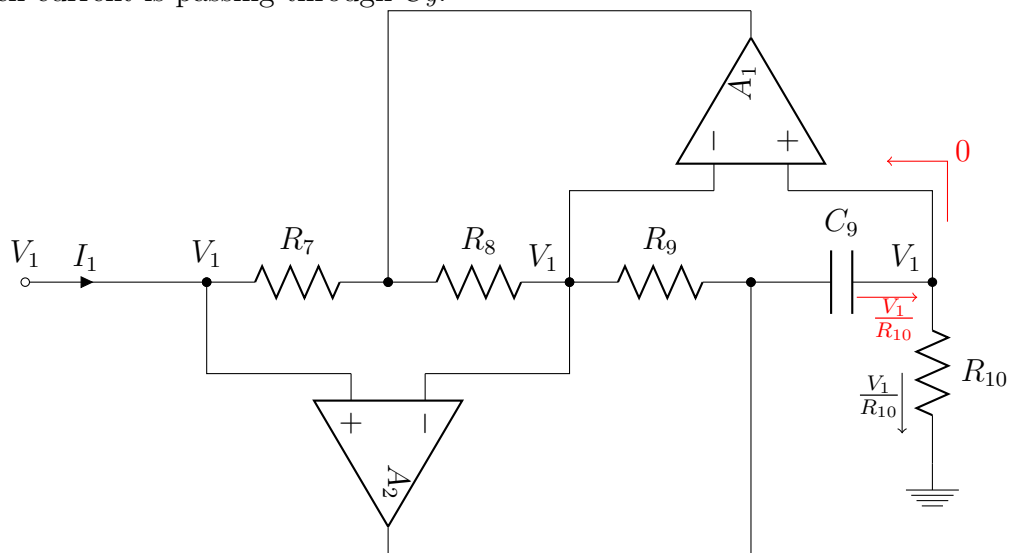

At this point we can apply Ohm's Law $(V = IR)$ to find the current
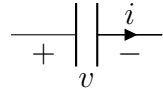
passing through R_{10}, $\frac{V_1}{R_{10}}$.

How much current is passing through the positive terminal of A_1? By the assumptions of an ideal op-amp, we know it's zero. From this, we know how much current is passing through C_9.

Now that we know the current passing through C_9, can we find out the voltage across C_9?

Recall, the terminal relationship of a capacitor is:

$$i(t) = C\frac{dv(t)}{dt}. \tag{5}$$

Here we will apply the Laplace transform to this terminal relationship and get:

$$I(s) = CsV(s) \tag{6}$$
$$or \tag{7}$$
$$V(s) = \frac{I(s)}{Cs} \tag{8}$$
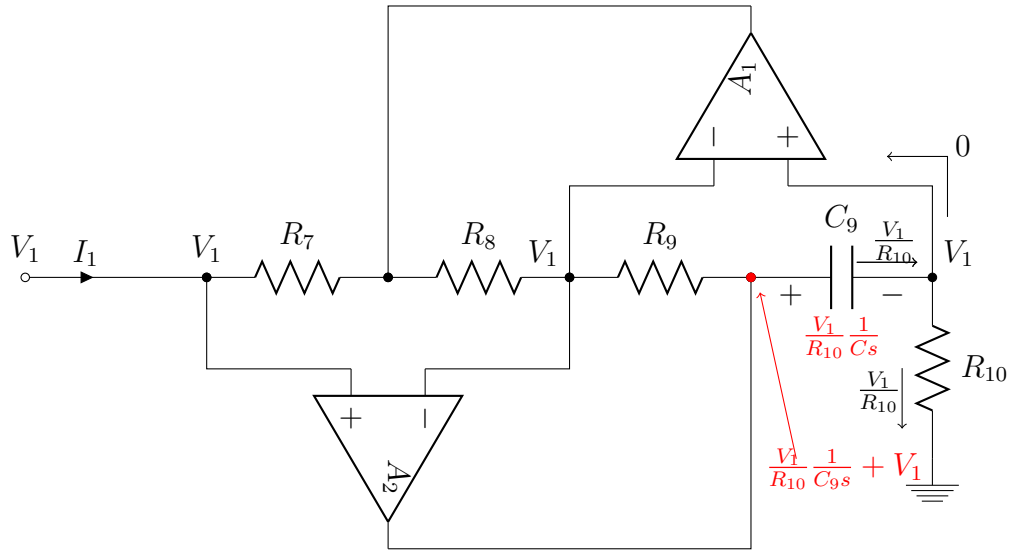
At this point we know enough to find out the voltage across C_9, and the voltage at the node between R_9 and C_9. Knowing the current going through C_9, we know the voltage across C_9 is

$$V_{C_9} = \frac{\frac{V_1}{R_{10}}}{C_9 s} \tag{9}$$
$$= \frac{V_1}{R_{10}}\frac{1}{C_9 s} \tag{10}$$

And the voltage at the node between R_9 and C_9 is the voltage across C_9 plus the voltage across R_{10} which is

$$V_{C_9} + V_1 = \frac{V_1}{R_{10}}\frac{1}{C_9 s} + V_1 \tag{11}$$

At this point you may realize that we now know the voltage across R_9 Implementing Ohm's Law, we also know the current:
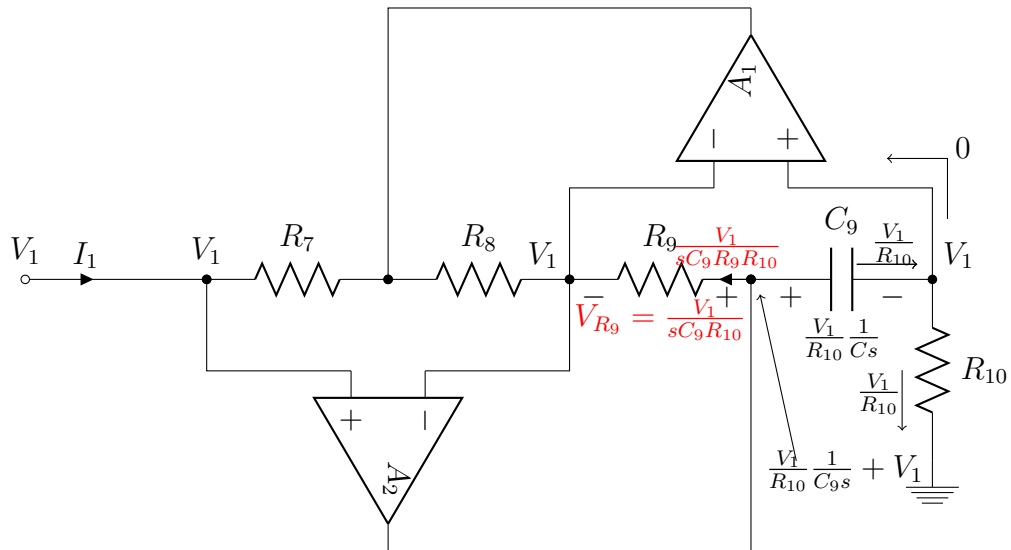
$$V_{R_9} = (\frac{V_1}{R_{10}}\frac{1}{C_9 s} + V_1) - V_1 \tag{12}$$
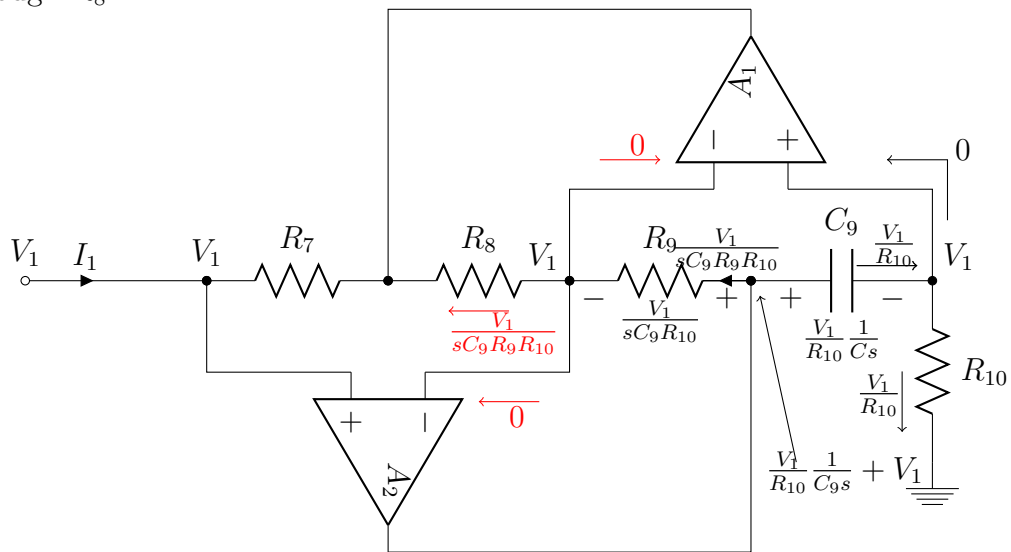
$$= \frac{V_1}{R_{10}}\frac{1}{C_9 s} \tag{13}$$

$$I_{R_9} = \frac{V_{R_9}}{R_9} \tag{14}$$

$$= \frac{\frac{V_1}{R_{10}}\frac{1}{C_9 s}}{R_9} \tag{15}$$

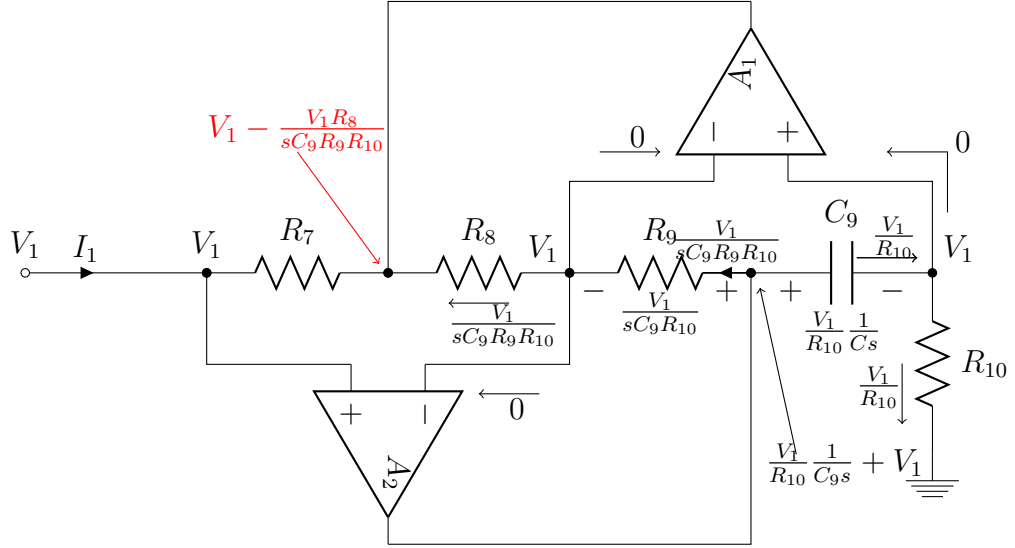$$= \frac{V_1}{sC_9 R_9 R_{10}} \tag{16}$$

At this point we go back to basics. We know that no current goes through the input terminals of the op-amps. Thus, we know that no current goes to the negative terminal of A_2 and no current goes into the negative terminal of A_1. Therefore, all the current from R_9 has nowhere to go except directly through R_8.

Knowing the current going through R_8 allows us to know the voltage across R_8, and the voltage at the node between R_7 and R_8.
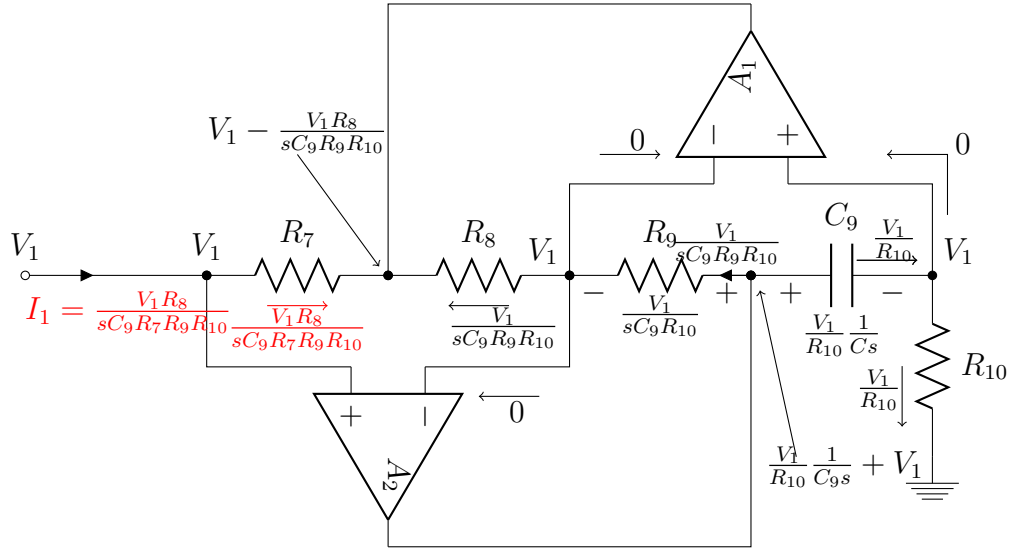
$$V_{R_8} = \frac{V_1 R_8}{sC_9 R_9 R_{10}} \tag{17}$$

From here we know the current going through R_7. Since no current goes through the positive terminal of A_2, we know I_1:

$$I_{R_7} = \frac{V_1 - [V_1 - \frac{V_1 R_8}{sC_9 R_9 R_{10}}]}{R_7} \tag{18}$$

$$= \frac{V_1 R_8}{sC_9 R_7 R_9 R_{10}} \tag{19}$$

$$I_1 = \frac{V_1 R_8}{sC_9 R_7 R_9 R_{10}} \tag{20}$$

$$V_1 - \frac{V_1 R_8}{sC_9 R_9 R_{10}}$$
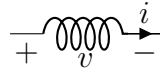
$$I_1 = \frac{V_1 R_8}{sC_9 R_7 R_9 R_{10}}$$

Now, we finally have the terminal relationship of the Antoniou Inductance-Simulation Circuit. Let's backtrack for a second and recall what our goal was... We wanted a circuit that had the same terminal relationship as an inductor.

$$v(t) = L\frac{di(t)}{dt} \tag{21}$$

$$\frac{V(s)}{I(s)} = sL \tag{22}$$

In our case

$$\frac{V_1(s)}{I_1(s)} = \frac{V_1}{\frac{V_1 R_8}{sC_9 R_7 R_9 R_{10}}} \tag{23}$$

$$\frac{V_1(s)}{I_1(s)} = \frac{sC_9 R_7 R_9 R_{10}}{R_8} \tag{24}$$

$$\tag{25}$$

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From here we can design an Antoniou Circuit with the inductance desired.

$$L_1 = \frac{C_9 R_7 R_9 R_{10}}{R_8} \tag{26}$$

$$\frac{V_1(s)}{I_1(s)} = sL_1 \tag{27}$$

This derivation is based on a compressed version found in [1, p. 1113].

References

[1] A. S. Sedra and K. C. Smith, *Microelectronic CIRCUITS, Fifth Edition.* Oxford University Press, 2007.